
Re: Is it true that a zero-inflated Gamma model is not possible

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--- "Swank, Paul R" <Paul.R.Sw...@UTH.TMC.EDU> wrote:

> I thought you couldn't have zeroes in a gamma distribution?

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That is correct. Thus, a zero-inflated gamma model is nothing more than modeling the zero values as a binary random r.v. and modeling the positive values employing the gamma distribution. Mathematically, a zero-inflated gamma could be described as

$$g(y) \sim B(1, f(x)) \quad , \quad g(y) = (y=0), y \geq 0$$

$$y \sim \text{gamma}(\text{theta}/h(x), \text{theta}), \quad y > 0$$

The first line indicates that we should compute and use as a response variable the function $g(y)$ which takes on the value 1 if $y=0$ and is 0 if $y>0$. (Note that any values of $y<0$ including values of y which are missing would become missing for $g(y)$.) This response is then modeled as a binary outcome. The function $f(x)$ is inverse logit function which constructs p from x . Employing a logit link, we would have

$$f(x) = \frac{\exp(b0_f + b1_f*x1 + b2_f*x2 + \dots)}{(1 + \exp(b0_f + b1_f*x1 + b2_f*x2 + \dots))}$$

The second line indicates that all of the positive values of y can be modeled as a gamma response. This is just what Amy has been doing up to this point, so there is nothing new here. The function $h(x)$ yields the expectation. Employing a log link, we would have

$$h(x) = \exp(b0_h + b1_h*x1 + b2_h*x2 + \dots)$$

The following NLMIXED code would allow one to fit a zero-inflated gamma distribution:

```
proc nlmixed data=mydata;
  parms b0_f=0 b1_f=0 b2_f=0 ...
        b0_h=0 b1_h=0 b2_h=0 ...
        log_theta=0;

  eta_f = b0_f + b1_f*x1 + b2_f*x2 + ...;
  p_yEQ0 = 1 / (1 + exp(-eta_f));

  eta_h = b0_h + b1_h*x1 + b2_h*x2 + ...;
  mu    = exp(eta_h);
  theta = exp(log_theta);
  r = mu/theta;
```

```

if y=0 then
  ll = log(p_yEQ0);
else
  ll = log(1 - p_yEQ0)
      - lgamma(theta) + (theta-1)*log(y) - theta*log(r) - y/r;

model y ~ general(ll);
predict (1 - p_yEQ0)*mu out=expect_zig;
predict r out=shape;
estimate "scale" theta;
run;

```

Given the single class predictor GROUP with three values Control, Intervention1, and Intervention2, we would write:

```

proc nlmixed data=mydata;
  parms b0_f=0 b1_f=0 b2_f=0
        b0_h=0 b1_h=0 b2_h=0
        log_theta=0;

  x1 = 0; if group='Intervention1' then x1=1;
  x2 = 0; if group='Intervention2' then x2=1;

  eta_f = b0_f + b1_f*x1 + b2_f*x2;
  p_yEQ0 = 1 / (1 + exp(-eta_f));

  eta_h = b0_h + b1_h*x1 + b2_h*x2;
  mu = exp(eta_h);
  theta = exp(log_theta);
  r = mu/theta;

  if y=0 then
    ll = log(p_yEQ0);
  else
    ll = log(1 - p_yEQ0)
        - lgamma(theta) + (theta-1)*log(y) - theta*log(r) - y/r;

  model y ~ general(ll);
  predict (1 - p_yEQ0)*mu out=expect_zig;
  predict r out=shape;
  contrast "Diff across trts in p(y=0)" b1_f, b2_f;
  contrast "Diff across trts in E(Y|y>0)" b1_h, b2_h;
  contrast "Diff across trts in E(Y)"
    exp(b0_h + b1_h)*(1 - 1/(1 + exp(b0_f + b1_f)))
    - exp(b0_h)*(1 - 1/(1 + exp(b0_f))),
    exp(b0_h + b2_h)*(1 - 1/(1 + exp(b0_f + b2_f)))
    - exp(b0_h)*(1 - 1/(1 + exp(b0_f)));
  estimate "scale" theta;
run;

```

Note that I added contrast statements in order to test the effect of treatment condition on each of the two components of this model: 1) the probability of a zero value, 2) the expectation of the nonzero values given the gamma distribution, and 3) the overall difference in expectations across the three treatment groups. I don't know the properties of the latter test. It may or may not be a good test for its purpose.

It should be pointed out that the only reason for estimating the zero-inflated gamma is in order to compute an expected response value (as output to the data set expect_zig) along with standard

errors of the expected value and/or to construct a test of whether there are differences across groups in the expected value of the mean response (not just the nonzero response values). If you do not want the expected value with standard errors or the particular test statistic which is obtained by the third contrast statement above, then it is easier to fit separate models estimating the probability of a zero value and then estimating the parameters of the gamma distribution.

A couple of final notes about a zero-inflated gamma model. The zero values are indicators of observations which are not generated under the gamma distribution. In the cost model, I would assume that these zero values indicate individuals who were service nonusers. Note, too, that the parameters of the gamma portion of the zero-inflated gamma are just the parameters of the gamma distribution which you can verify by fitting the NLMIXED code given above and then fitting a gamma response model using GENMOD with log link.

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