# Evaluating benefit-risk, handling missing data and a universal sample size formula for clinical trials 

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## As the Title Says...

- Benefit-Risk evaluation
- Handling missing data
- Universal (non-parametric) sample size formula for parallel group trials

(this one is joint with Lu Tian of Stanford)

## Main Idea

- Organize endpoints by importance
- Compare pairs of patient data on most important endpoint Possible outcomes: better (+1), worse (-1), tie (0) (Mann-Whitney U test)
- Comparison is a tie? Compare next-most important endpoint Comparisons occur at minimum of follow-up times (Finkelstein-Schoenfeld)
- Calculate $\left.\widehat{\boldsymbol{\theta}}=\frac{\operatorname{Prob}(\mathrm{A}>\mathrm{B})}{\operatorname{Prob}(\mathrm{A}} \mathrm{B}\right)$
(">" means better than, "<" worse than. A and B are randomly selected patients from Groups 1 and 2)


## Examples

Endpoint 1: time to death. Endpoint 2: frequency of hospitalizations

## Who did better?



Case $2\left\{\begin{array}{lll}\text { A Hosp. } & \text { Hosp. } \\ \text { B } \xrightarrow[\text { Hosp. }]{\text { Hlive }}\end{array}\right.$

## Benefit-Risk

## Current Approach

- Drug approval depends on benefit-risk evaluation
- Benefit: efficacy endpoints, formally evaluated
- Risk: subjective evaluation of adverse events

Combined evaluation is subjective
Side note: term 'benefit-risk' is not symmetric
Benefit: gain that will be accrued
Risk: harm that may be experienced
Neutral term: benefit-harm

## Example: Idiopathic Pulmonary Fibrosis (Ofev)

Primary efficacy endpoint: Annual rate of decline in forced vital capacity (FVC)
B inpulsis-1

No. of Patients
Nintedanib
292
202198200
194
284
274
250
Placebo
192
187
165

## Efficacy: IPF (Ofev)

Key secondary: time to first acute exacerbation

No. of Patients
Nintedanib Placebo
$302 \quad 304$ $\begin{array}{ll}290 & 288 \\ 197 & 197\end{array}$ $\qquad$ 275


Figure 2. Time to First Investigator-Reported Acute Exacerbation in INPULSIS-1 and INPULSIS-2.

## Efficacy: IPF (Ofev)

Other secondary: FVC percent predicted and survival
Table 2. Secondary Lung-Function End Points at Week 52.

| End Point | INPULSIS-1 |  |  |  | FVC percent predicted: Similar for INPULSIS-2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Nintedanib $(\mathrm{N}=307)$ | Placebo $(N=204)$ | Difference, Nintedanib vs. Placebo (95\% CI) | P Value |  |
| Adjusted absolute mean change from baseline in FVC - ml | -95.1 | -205.0 | $\begin{gathered} 109.9 \\ (71.3 \text { to } 148.6) \end{gathered}$ | <0.001 |  |
| Adjusted absolute mean change from baseline in FVC - \% of predicted value | -2.8 | -6.0 | $\begin{gathered} 3.2 \\ (2.1 \text { to } 4.3) \end{gathered}$ | <0.001 |  |

(Table S8 in the Supplementary Appendix). The

Survival proporion of pationto who dicel frome any catos over the 52-week treatment period was $5.5 \%$ in the nintedanib group and $7.8 \%$ in the placebo
group (hazard ratio in the nintedanib group, $0.70 ; 95 \% \mathrm{CI}, 0.43$ to $1.12 ; \mathrm{P}=0.14$ ) (Fig. S 8 in the

## Safety: IPF (Ofev)

The most frequent serious adverse reactions reported in patients treated with OFEV, more than placebo, were bronchitis ( $1.2 \%$ vs. $0.8 \%$ ) and myocardial infarction ( $1.5 \%$ vs. $0.4 \%$ ). The most common adverse events leading to death in patients treated with OFEV, more than placebo, were pneumonia ( $0.7 \%$ vs. $0.6 \%$ ), lung neoplasm malignant ( $0.3 \%$ vs. $0 \%$ ), and myocardial infarction ( $0.3 \%$ vs. $0.2 \%$ ). In the predefined category of major adverse cardiovascular events (MACE) including MI, fatal events were reported in $0.6 \%$ of OFEVtreated patients and $1.8 \%$ of placebo-treated patients.

Adverse reactions leading to discontinuation were reported in $21 \%$ of OFEV-treated patients and $15 \%$ of placebo-treated patients. The most frequent adverse reactions that led to discontinuation in OFEV-treated patients were diarrhea $(5 \%)$, nausea $(2 \%)$, and decreased appetite ( $2 \%$ ).

## Safety: IPF (Ofev)

Summary of Adverse Events (portion of table reproduced) Multiple occurrences usually not included

Table 1 Adverse Reactions Occurring in $\geq 5 \%$ of OFEV-treated Patients and More Commonly Than Placebo in Studies 1,2, and 3

| Adverse Reaction | OFEV, 150 mg <br> $\mathbf{n = 7 2 3}$ | Placebo <br> $\mathbf{n = 5 0 8}$ |
| :--- | :---: | :---: |
| Gastrointestinal disorders | $62 \%$ | $18 \%$ |
| Diarrhea | $24 \%$ | $7 \%$ |
| Nausea | $15 \%$ | $6 \%$ |
| Abdominal pain ${ }^{\text {a }}$ | $12 \%$ | $3 \%$ |
| Vomiting |  |  |
| Hepatobiliary disorders | $14 \%$ | $3 \%$ |
| Liver enzyme elevation $^{\text {b }}$ |  |  |

How to quantify benefit-risk?

## Proposal

- Before unblinding, specify important efficacy and safety endpoints

Arrange endpoints by priority

## Example

death, acute exacerbations, SAE1, FVC, SAE2
(allows for inclusion of multiple occurrences of events)

- First calculate $\widehat{\boldsymbol{\theta}}$ and C.I. for binary version of primary endpoint
- Next calculate $\widehat{\boldsymbol{\theta}}$ and C.I. sequentially


## Analysis

- $\widehat{\operatorname{Prob}}(\mathrm{A}>\mathrm{B})=$

$$
\frac{1}{n m} \sum \sum \mathbf{I}\left(\text { Group }_{\boldsymbol{j}}>\operatorname{Group}_{\boldsymbol{i}}^{\boldsymbol{i}}\right)
$$

Similarly get $\widehat{\operatorname{Prob}}(\mathrm{A}<\mathrm{B}) . \quad \widehat{\boldsymbol{\theta}}$ is ratio of probabilities

- $95 \%$ C.I. $=\exp (\ln \widehat{\theta} \mp 1.96 \sqrt{v a r})$

No variance formula.
Bootstrap recommended

- $\operatorname{var} \approx \frac{4}{3(n+m) k(1-k)} \times \frac{\left(1+p_{\text {tie }}\right)}{\left(1-p_{t i e}\right)}$
- $n$ and $m$ are group sizes. $k$ is proportion in group 1
- $p_{\text {tie }}=\operatorname{Prob}($ tie $)$


## Relevant Literature

1. Mann-Whitney (Annals Math Stat, 1947) - key idea
2. Finkelstein and Schoenfeld (Stat Med 1999)
3. Buyse (Stat Med 2007)
4. Pocock et al (Eur H J 2012)
5. Yu and Ganju (manuscript under review. Var formula doesn't require individual level data)
6. Evans and Follmann. Stat Biopharm Res 2016 (on Benefit-Risk)

## Formula Applied to Real Data



Published result required individual level data.

Formula (approximation) requires summary level data

Figure from: Sample size formula for a win ratio endpoint. Yu and Ganju, Manuscript under review.

## Benefit-Risk Evaluation: Made Up Example

| FVC (binary) at week 52 |
| :--- |
| (made binary to create ties) |$\quad \hat{\theta}=1.50 \quad 95 \% \mathrm{CI}: 1.25,1.75$

Benefit-Risk Assessment

| Endpoint | $\widehat{\theta}$ | $95 \% \mathrm{C}$. |
| :--- | :---: | :---: |
| Death | 1.03 | $0.15,1.91$ |
| + Acute exacerbation | 1.10 | $0.49,1.57$ |
| + SAE1 | 1.02 | $0.62,1.43$ |
| + FVC at week 52 (binary) | 1.35 | $1.13,1.57$ |
| + SAE2 | 1.15 | $1.03,1.45$ |

Null: $\theta=1$
B-R favorable if $\theta>1$
FVC binary: week 52 value within $5 \%$ of baseline value

## Handling of Missing Data

## Convention

- One primary method, plus
'Sensitivity' analyses, sometimes several
Results will vary
- Unknown: Which result should the public accept?

The NEW ENGLAND
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HIGHLIGHTS OF PRESCRIBING INFORVIATION
These highlights do not include all the information needed to use

What principle should guide us?

## Cardiovascular Trial: ATTR-ACT

A Change from Baseline in 6-Minute Walk Test


- 6MWT, secondary endpoint
- Missing at M30: 49\%
- MMRM primary
- Sensitivity analyses (pattern mixture models)

Maurer et al NEJM 2018

## CV Trial: ATTR-ACT

Mortality
B Analysis of All-Cause Mortality

CV hosp.



No. at Risk (cumulative no. of events)
Pooled tafamidis $\quad 264(0) \quad 259(5) \quad 252(12) 244(20) \quad 235(29) 222(42) 216(48) 209(55) 200(64) 193(71) 99(78) \quad 0(78)$


Benefit on mortality and \# of CV hosp. (primary endpoint). Contributes to missing and 'missing' 6MWT at M30.

## CV Trial: ATTR-ACT

FDA statistical review
Table 8: Comparison of Main Analysis and Sensitivity Analysis Results on 6MWD (ITT)

|  | LS means | $95 \% \mathrm{Cl}$ |
| :--- | :---: | :---: |
| Main analysis | 75.7 | $(57.6,93.8)$ |
| Pattern mixture analysis | 61.5 | $(44.4,78.5)$ |

[Source: Reviewer's table]
Variance of analysis? -

Suppose the mortality benefit was even greater than what was observed. Would the pattern mixture result move closer to the MMRM result or away from it?

## Estimands

ICH E9 (R1) addendum

## Composite variable strategies

This relates to the variable of interest (see A.3.3.). An intercurrent event is considered in itself to be informative about the patient's outcome and is therefore incorporated into the definition of the variable. For example, a patient who discontinues treatment because of toxicity may be considered not to have been successfully treated. If the outcome variable was already success or failure,

Proposed method: prioritize reason for missing data and include time when missing data occurred

## Prioritization of Reasons

- Both patients have month 30 6MWT data

Compare changes from baseline

## Who did better?



- Only one patient has month 30 6MWT data


## Who did better?



Big decline, but A still better than $B$

## Prioritization of Reasons

- Both patients have missing data for the same reason


## Who did better?



## Prioritization of Reasons

- Both patients have missing data for different reasons


## Who did better?

Case 1 $\begin{cases}A-M 23, \text { withdrew consent } \\ B-M 20, A E\end{cases}$


Case $2 \begin{cases}A & \mathrm{M} 20, \text { withdrew consent } \\ B \quad \mathrm{M} 23, \mathrm{AE}\end{cases}$

## Prioritization of Reasons

- Both patients have missing data for different reasons


## Who did better?

Case 1 $\begin{cases}A \longrightarrow & M 23, \text { death } \\ B & M 20, A E\end{cases}$

Case $2 \begin{cases}A & \text { M20, death } \\ B & M 23, A E\end{cases}$

## Analysis

As before
$\widehat{\operatorname{Prob}}(\mathbf{A}>\mathbf{B})=\quad \frac{1}{n m} \sum \sum \mathbf{I}\left(\operatorname{Group}_{\boldsymbol{j}}>\operatorname{Group}_{\boldsymbol{i}}^{\boldsymbol{i}}\right)$
$\widehat{\boldsymbol{\theta}}=\frac{\widehat{\operatorname{Prob}}(\mathrm{A}>\mathrm{B})}{\widehat{\operatorname{Prob}}(\mathrm{A}<\mathrm{B})}$
$95 \%$ C.I. for $\widehat{\theta}: \exp (\ln \widehat{\theta} \mp 1.96 \sqrt{v a r})$

## Missing Data and Prioritization of Reasons

- Endpoint is now a combination endpoint
- Rules for handling missing data transparent and easily understood
- Loss of power possible compared with model-based methods
- Idea extends to other kinds of endpoints


## Conclusions (Benefit-Risk, Missing Data)

- Benefit-Risk
- Pairwise comparisons of hierarchically arranged endpoints provides a framework for evaluation of whether benefit > risk, and if so, by how much
- For drugs treating the same condition, can compare the benefit-risk
- Missing data
- Prioritizes reasons (including timing) for missingness. Non-parametric solution to the missing data problem


## Universal (non-parametric) Sample Size Formula for Parallel Group Trials

## Mann-Whitney U Test

- Nonparametric test of $H_{0}: \operatorname{Pr}(A>B)=\operatorname{Pr}(A<B)$, where $A$ and $B$ are randomly selected values from two populations

$$
U=\sum_{i=1}^{n} \sum_{j=1}^{m} S\left(A_{i}, B_{j}\right)
$$

Under $\mathrm{H}_{0}$ (in the absence of tied ranks),

$$
\begin{aligned}
& E(U)=0 \\
& \operatorname{Var}(U)=\frac{n m(n+m+1)}{3} \\
& Z=\frac{U-E(U)}{\sqrt{\operatorname{Var}(U)}}
\end{aligned}
$$

with

$$
S(A, B)=\left\{\begin{array}{cc}
1, & \text { if } B<A \\
0, & \text { if } B=A \\
-1, & \text { if } B>A
\end{array}\right.
$$

Reject $\mathrm{H}_{0}$ if $|Z| \geq Z_{1-\frac{\alpha}{2}}$

## General Sample Size Formula for MannWhitney U Test Is Not Available

- Only available for special cases, e.g.
- Continuous endpoint (Noether 1987)
- Ordinal endpoint (Whitehead 1993; Zhao et al. 2008)
- Recurrent event endpoint under equal duration of follow up


## Inputs to Universal Sample Size Formula for Parallel Group Trials

- $\mathrm{H}_{0}: \theta=1$

$$
\mathrm{H}_{1}: \theta>1
$$

$$
\theta=\frac{\operatorname{Pr}(A>B)}{\operatorname{Pr}(A<B)}
$$

- Inputs to universal sample size formula are:
-Allocation ratio, $\theta$ under $\mathrm{H}_{1}, \alpha, \beta$
- Probabilities of transitive relationships (under $\mathrm{H}_{0}$ )
- Probabilities of intransitive relationships (under $\mathrm{H}_{0}$ )


## Transitive Relationships



EP1 - Time to Death EP2 - Time to first Hospitalization

$$
\mathrm{A}>\mathrm{B}, \quad \mathrm{~B}>\mathrm{C}, \quad \mathrm{~A}>\mathrm{C}
$$

## Intransitivity - Severe



EP1 - Time to Death EP2 - Time to first Hospitalization

$$
\mathrm{A}>\mathrm{B}, \quad \mathrm{~B}>\mathrm{C}, \quad \text { but } \mathrm{C}>\mathrm{A}!
$$

## Intransitivity - Moderate



EP1 - Time to Death EP2 - Time to first Hospitalization

$$
A>B, \quad B=C, \quad \text { but } C>A!
$$

## Intransitivity - |Mild



EP1 - Time to Death

$$
A=B, \quad B=C, \quad \text { but } C>A!
$$

## Different Types of Intransitivity

Severe
$A>B, B>C$, but $C>A$

Moderate
$A>B, B=C$, but $C>A$
Mild
$A=B, B=C$, but $C>A$

## Conditions Giving Rise to Intransitivity

- Variable lengths of follow up
and
- Endpoint
- Survival endpoint (mild intransitivity)
- Recurrent event endpoint (any kind of intransitivity)
- Hierarchical combination of endpoints (any kind of intransitivity)
- Etc.


## Derivation of Null Variance

- Null variance involves comparisons of a triplet of patients
- Each pairwise comparison has three possible outcomes: win, loss, or tie
- As a result, there are $3^{3}=27$ possible scenarios
- 13 are transitive scenarios; 14 are intransitive scenarios


## Seven Distinct Probabilities

- Transitive relationships
- $\mathrm{A}>\mathrm{B}>\mathrm{C}$
- $A>B=C$
- $A=B>C$
- $A=B=C$
- Intransitive relationships
- $A>B, B>C, C>A$ (severe)
- $A>B, B=C, C>A$ (moderate)
- $A=B, B=C, C>A$ (mild)


## Null Variance Formula

- $\operatorname{Var}(\widehat{\operatorname{Pr}}(A>B)-\widehat{\operatorname{Pr}}(A<B)) \approx \frac{n+m}{n m}\left(\frac{p_{1}+p_{2}+p_{3}}{3}-p_{5}-\frac{p_{6}}{3}\right)$

$$
=\frac{n+m}{3 n m}\left[1-p_{4}-\left(4 p_{5}+2 p_{6}+p_{7}\right)\right]
$$

where

$$
\begin{aligned}
& p_{1}=\operatorname{Pr}(A>B>C) \\
& p_{2}=\operatorname{Pr}(A>B=C) \\
& p_{3}=\operatorname{Pr}(A=B>C) \\
& p_{4}=\operatorname{Pr}(A=B=C)=\operatorname{Pr}(3-\text { way tie }) \\
& p_{5}=\operatorname{Pr}(A>B, B>C, C>A)=\operatorname{Pr}(\text { severe intransitivity }) \\
& p_{6}=\operatorname{Pr}(A>B, B=C, C>A)=\operatorname{Pr}(\text { moderate intransitivity }) \\
& p_{7}=\operatorname{Pr}(A=B, B=C, C>A)=\operatorname{Pr}(\text { mild intransitivity) }
\end{aligned}
$$

## Special Cases

$$
\frac{n+m}{3 n m}\left[1-p_{4}-\left(4 p_{5}+2 p_{6}+p_{7}\right)\right]
$$

- No tie, no intransitivity

$$
\operatorname{Var}(\widehat{\operatorname{Pr}}(A>B)-\widehat{\operatorname{Pr}}(A<B)) \approx \frac{n+m}{3 n m}
$$

- No intransitivity (ties okay)

$$
\operatorname{Var}(\widehat{\operatorname{Pr}}(A>B)-\widehat{\operatorname{Pr}}(A<B)) \approx \frac{n+m}{3 n m}\left(1-p_{4}\right)
$$

- Survival endpoint

$$
\operatorname{Var}(\widehat{\operatorname{Pr}}(A>B)-\widehat{\operatorname{Pr}}(A<B)) \approx \frac{n+m}{3 n m}\left(1-p_{4}-p_{7}\right)
$$

## Sample Size Formula for Parallel Group Trials

$$
\begin{gathered}
\operatorname{Var}(\log (\hat{\theta})) \approx \frac{4}{(1-\operatorname{Pr}(2-\text { way tie }))^{2}} \operatorname{Var}(\widehat{\operatorname{Pr}}(A>B)-\widehat{\operatorname{Pr}}(A<B)) \\
=\frac{\sigma^{2}}{N} . \quad(N=n+m) \\
N=\frac{\sigma^{2}\left(Z_{1-\alpha}+Z_{1-\beta}\right)^{2}}{\log ^{2}(\theta)}
\end{gathered}
$$

## Conclusions

- Mann-Whitney $U$ test has been around for a long time, but until now a general sample size formula has not been available
- Hierarchically combined endpoint is getting popular, sample sizes are calculated via simulations which are time-consuming and not straightforward
- To use the proposed formula, we need to build experience with the component probabilities:
- Probability of a 3-way tie
- Probabilities of intransitive outcomes


## References

- Noether, G. E. (1987). Sample size determination for some common nonparametric tests. Journal of the American Statistical Association, 82(398), 645-647.
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- Zhao, Y. D., Rahardja, D., \& Qu, Y. (2008). Sample size calculation for the Wilcoxon-Mann-Whitney test adjusting for ties. Statistics in medicine, 27(3), 462-468.

