Introduction to Exploratory Factor Analysis (EFA)

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Methods Core
&
Visiting Professors Program

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Overview: EFA is one form of the common factor model

Conceptually, what is a common factor model?

Indirect observation

Some constructs are not directly observable
attitudes, intelligence, economic strength, top quark

Indirectly measured constructs are sometimes called *latent* variables
. Latent variables are 'everywhere' (physics, medicine, economics)

Latent variables often are identified via
multiple, fallible, observed—or manifest—variables

A measurement model relates latent variables to manifest variables.
That is, the latent variables are hypothesized to directly cause
responses to corresponding manifest variables

With multiple manifest variables per latent variable, the measurement
model can be empirically evaluated, via common factor analysis

What does 'common' mean?

What are the goals of common factor model?

Assess a form of validity, i.e., construct validity.
Do the items measure the hypothesized constructs?

Represent a set of observed variables (or items)
by a more parsimonious set of related constructs
(AKA common factors, latent variables)

Provide empirical justification for creating
composite scores, or 'scale scores,'
which are more reliable than individual item scores

The common factor model is >115 years old (Spearman 1904*)*

Conceptual example: Consumer confidence

Suppose I want to measure two dimensions of consumer confidence

Economic expectations for the upcoming 6-months

. Business conditions (1 = worse; 2 = same; 3 = better)

. Employment (1 = fewer jobs; 2 = same; 3 = more jobs)

. Income (1 = decrease; 2 = same; 3 = increase)

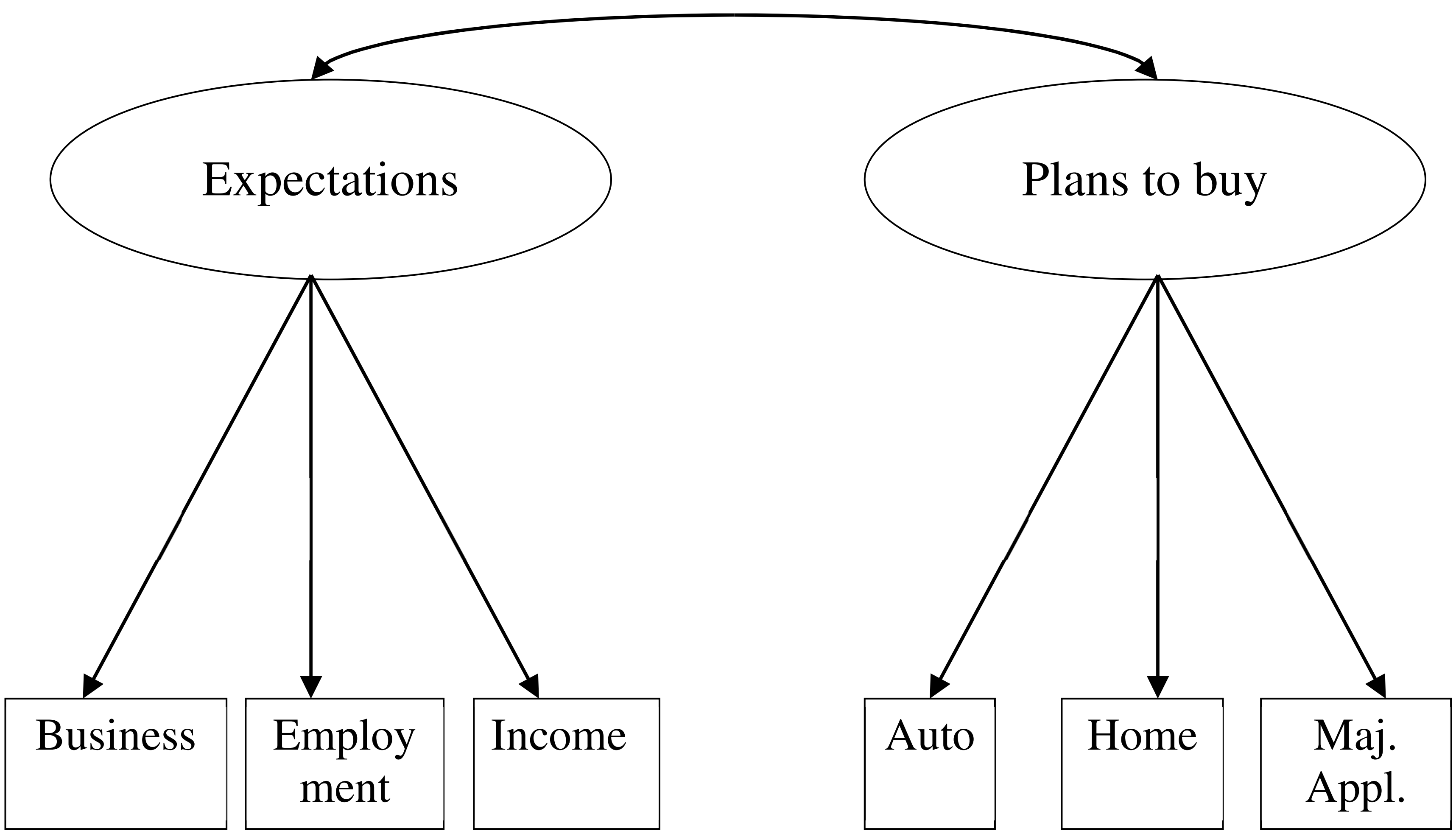
Personal purchase plans within 6-months

. Automobile

. Home

. Major appliances

Consumer confidence: Common factor configuration



(define single- and double-headed arrows)

Consumer confidence: *Made-up* common factor model

A generic representation of a factor pattern matrix
with 2 common factors and 6 manifest variables

	Expectations	Plans to buy
business	.67	.12
employment	.54	.11
income	.55	.07
auto	.05	.77
house	.09	.89
major appl.	.10	.57

The factor pattern matrix holds estimated correlations between
latent and manifest variables

The latent variables are estimated from the observed data
. latent variables are unobserved, so their scaling is arbitrary

Correlations between latent and manifest variables aid interpretation

Q: Is the interpretation consistent with the motivating hypotheses?

Wait a minute…

How is it possible to estimate the relationship between something measured (items) and something not measured (factors)?

Start with input data

The input data for a factor analysis are usually the observed correlations or covariances among the observed items

Estimate factor loadings for your hypothesized model (iterative search)

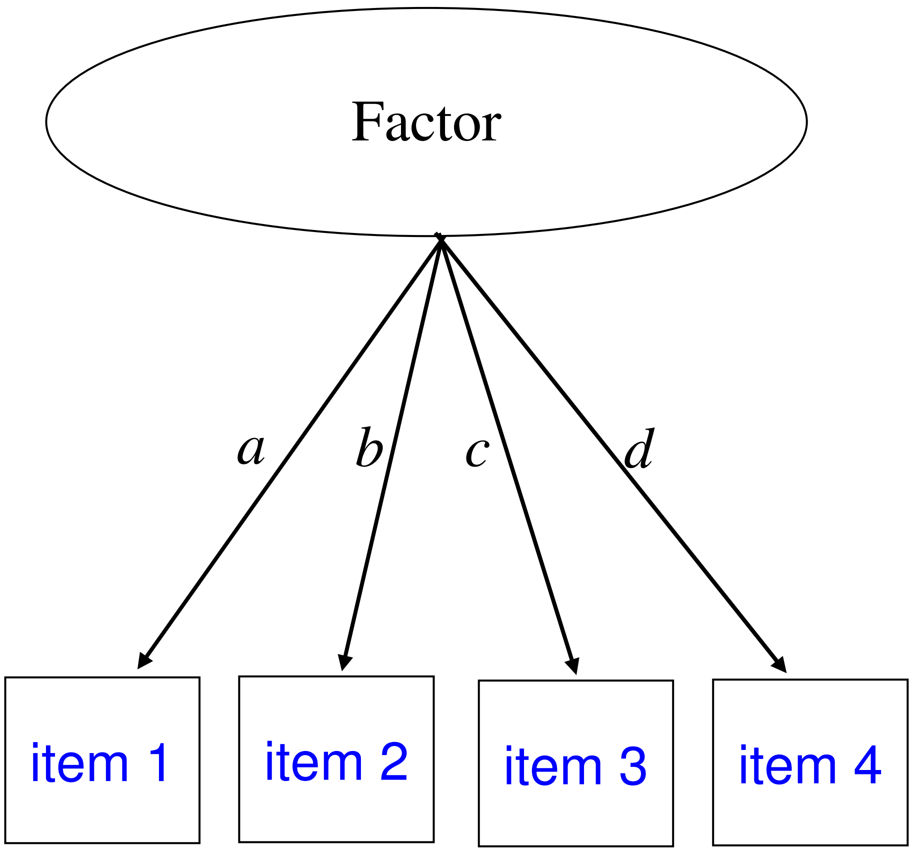
A well-fitting factor model and estimates can be used to accurately reproduce the input data

Compare the model-reproduced data to the original data

Good correspondence between the two suggests that the model has 'good fit' and we have more confidence in the model and estimates

Relationship between standardized factor loadings & item correlations

Factor model & loading estimates



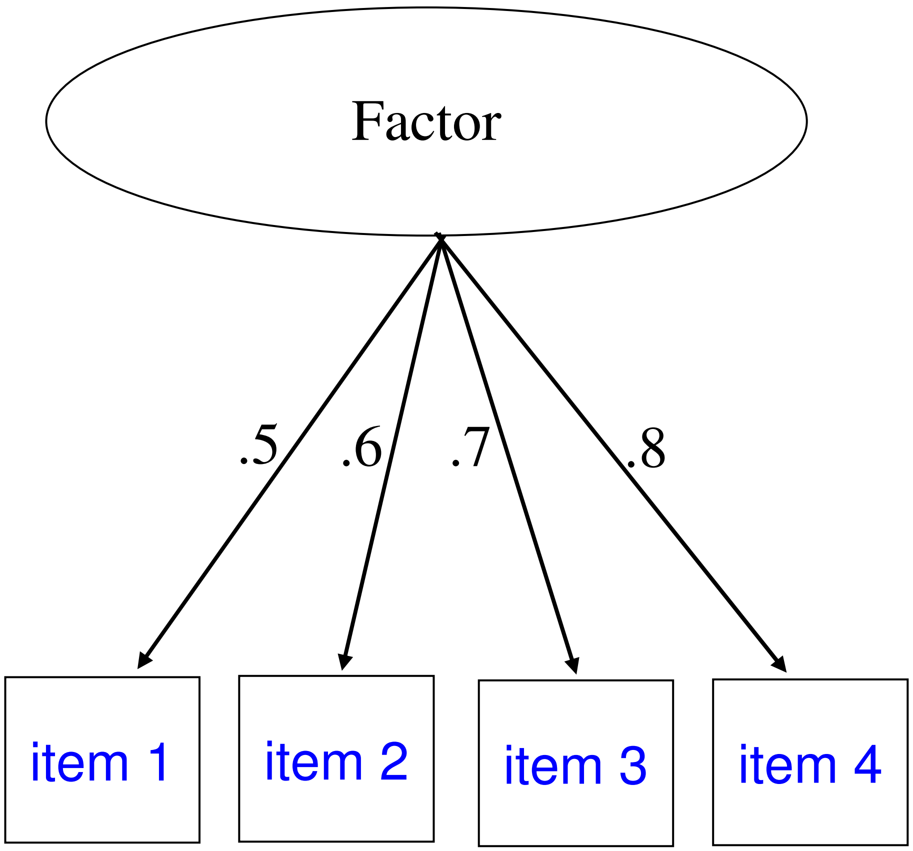
Model-implied item correlations

	item 1	item 2	item 3	item 4
item 1	1.0			
item 2	$a \times b$	1.0		
item 3	$a \times c$	$b \times c$	1.0	
item 4	$a \times d$	$b \times d$	$c \times d$	1.0

. 4 factor loadings (a, b, c, d) attempt to explain 6 inter-item corrs.

Relationship between standardized factor loadings & item correlations

Factor model & loading estimates



Model-implied item correlations

	item 1	item 2	item 3	item 4
item 1	1.0			
item 2	.30	1.0		
item 3	.35	.42	1.0	
item 4	.40	.48	.56	1.0

Empirical question

Do the model-implied correlations approximate the observed correlations?
There are chi-square goodness-of-fit tests to address that question.

Again: Implications of empirical support for a common factor model

Demonstration of construct validity:
Do the items measure what they are hypothesized to measure?

More parsimonious representation of information captured by items

Provides empirical justification for creating
composite scores, or 'scale scores,'
which are more reliable than individual item scores

Optimally, the <u>configuration</u> of a common factor (AKA measurement) model is specified a priori

The configuration specifies a set of common factors (latent variables). Each is hypothesized to cause responses to specific subsets of items (manifest variables).

It should be based upon theory, or previous empirical findings

But…

. What if the hypothesized measurement model isn't supported?

. Or, what if there is no a priori measurement model to test?

One option…

. Exploratory factor analysis (EFA)

The goal of EFA is to uncover the measurement model

Introduction: Steps in EFA

(1) initial choice of items to factor analyze

(2) respondent sampling and data collection

(3) compute matrix of inter-item correlations or covariances

(4) specify number of factors

(5) specify method of factor extraction

(6) specify method of factor rotation

(7) interpret/assess model:

. Is the model substantively appealing?

. Is the specified number of factors reasonable?

. Are any items questionable?

. Possibly re-specify # of factors and/or drop items and re-fit model

Introduction: Step 1. Choose items to factor analyze

Choose

. constructs (common factors, latent variables) to be measured

. items (observed, manifest variables) representing each construct

In instrument development, choosing items often begins with crafting a superset of items and choosing from among them.

Introduction: Step 2. Sampling and data collection

Factor analysis requires that responses to a set of 'items' are collected from a sample of (usually) individuals

The sampling method should select individuals who are representative of the targeted population, i.e., generalizability of findings

Introduction: Step 3. Compute matrix of inter-item correlations or covariances

Inter-item correlations between 4 made-up items

. Start with collected data on items—the observed/manifest variables

. Estimate so-called 'reduced,' item correlation or covariance matrix

'Full' Correlation Matrix (diagonal=1)

	item *a*	item *b*	item *c*	item *d*
item *a*	1.0			
item *b*	.30	1.0		
item *c*	.35	.42	1.0	
item *d*	.40	.48	.56	1.0

'Reduced' Correlation Matrix

	item *a*	item *b*	item *c*	item *d*
item *a*	h_a^2			
item *b*	.30	h_b^2		
item *c*	.35	.42	h_c^2	
item *d*	.40	.48	.56	h_d^2

Diagonal of the reduced matrix holds communality estimates, h_i^2
i.e., the variance of each item that is explained by the factors

An Aside. Exploratory Factor Analysis (EFA) versus Principal Components Analysis (PCA)

Mathematically, EFA and PCA are very similar

.EFA decomposes the reduced item correlation (covariance) matrix

.PCA decomposes the full item correlation (covariance) matrix

However, EFA and PCA are entirely different

. EFA is an end in itself

. EFA primary goal: understand latent structure, validation

. EFA secondary goal: data reduction, composite measures

. EFA: causal model to explain cognitive/psychological phenomena

. PCA is a means to an end

. PCA primary goal: data reduction, composite measures

. PCA: data dimensionality and correlation structure represent nuisance factors to be tamed

Introduction: Step 4. Specify number of factors

Exploratory methods to empirically determine the number of factors

. Eigenvalue > 1.0 rule

. Scree plot of eigenvalues

. χ^2 tests of model fit and fit indices (requires ML factor analysis)

- ML chi-square
- Fit indices: Comparative Fit Index (CFI; Bentler, 1999)

  Bentler, P. M. (1990). Comparative fit indices in structural models. *Psychological Bulletin*, 107, 238-246.

So, what is an eigenvalue? One possible definition

The variance explained by the corresponding principal component (i.e., a somewhat advanced topic in matrix algebra)

Bizarrely, both the eigenvalue>1 and scree plot EFA approaches to choosing the number of common factors to extract can be construed as resting upon results of a PCA!

Introduction: Step 4. Specify number of factors

Comparative Fit Index (CFI)

CFI requires use of ML EFA to obtain model chi-square values

$\text{CFI} = 1 - \left(\chi_k^2 - d_k\right)/(\chi_0^2 - d_0)$, where

χ_k^2 and d_k are the chi-square & *df* from the target model with *k* factors

χ_0^2 and d_0 are the chi-square & *df* from a model w/ no common factors

Most ML EFA factor analysis programs will output both chi-squares

You can write code to calculate CFI, or calculate by hand

CFI ranges from 0-1.
IMO, CFI < 0.95 often suggests more factors should be extracted

Among the empirical options for choosing # of factors, I use CFI

Introduction: Step 5. Choose factor extraction method

Common factor analysis

Decomposes *common* variation in the item correlation matrix

I.e., analysis of the reduced correlation (or covariance) matrix

Some approaches

. Two step: Squared multiple correlations, then extraction

. Iterative simultaneous parameter and communality estimation

. Several approaches including ML EFA

Extraction, per se, is an application of matrix algebra

I use ML EFA because it allows calculation of CFI

Introduction: Step 6. Specify method of factor rotation

Extracted factors are uncorrelated and are usually difficult to interpret

Factor rotation hopefully allows for easier interpretation

Orthogonal rotation: uncorrelated factors

. VARIMAX (all factor analysis programs)

. many others

Oblique rotation: correlated factors

. PROMAX (SAS)

. Harris-Kaiser (SAS)

. Direct Oblimin (SPSS)

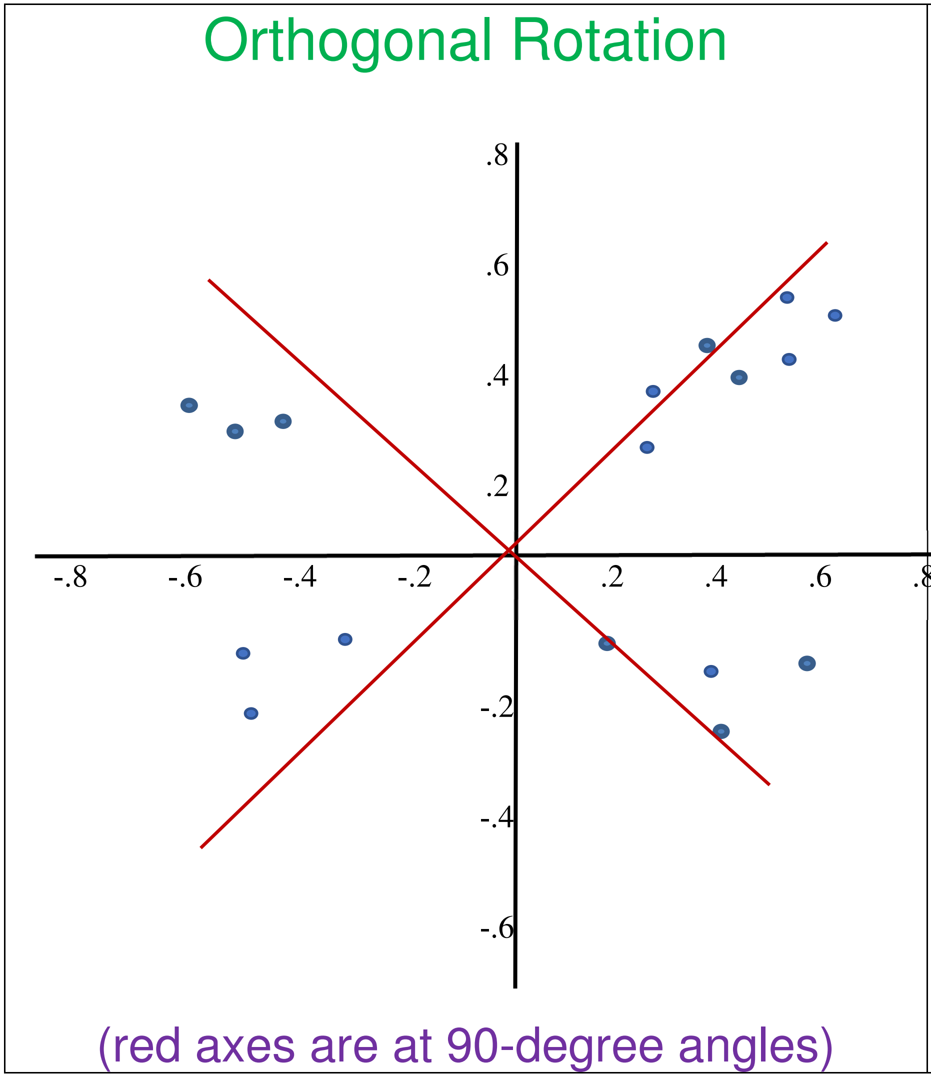
. many others

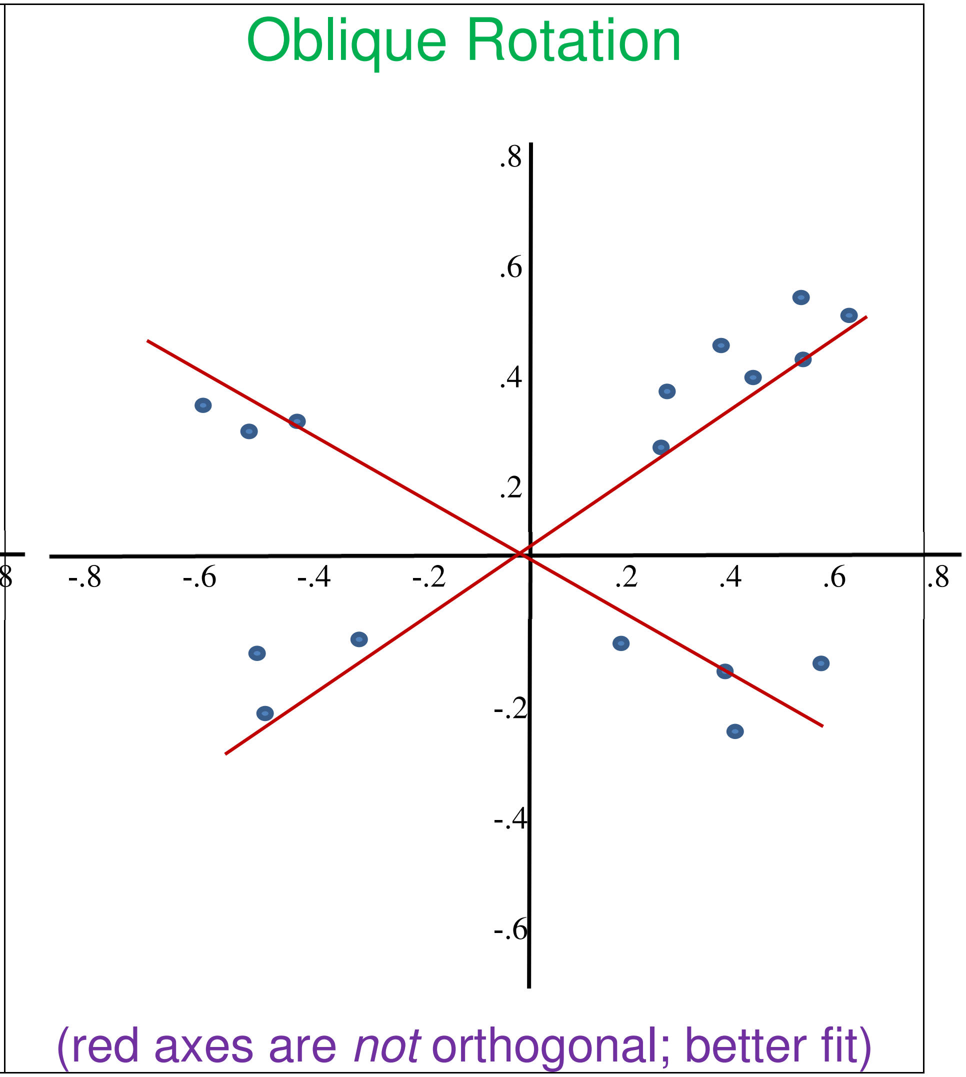
Unless you <u>absolutely</u> require orthogonal rotation, choose oblique

Introduction: Step 6. Factor rotation

. Black Axes are extracted factors; Blue dots represent items

. Red Axes are Rotated Factors





Example application

NHANES 1982-84 Epi Follow-up

Center for Epidemiologic Studies Depression scale (CES-D)

. White men aged 50+ with complete data on all 20 CES-D items
. *N* = 2004

The items of the CES-D are generally believed to represent 4 factors

factor	items
depressive affect	*blues, depressed, failure, fearful, lonely, cry, sad*
somatic symptoms	*bothered, appetite, mind, effort, sleep, talk, get going*
inter-personal	*unfriendly, dislike*
positive affect	*good, hopeful, happy, enjoy*

Example: Steps 1, 2, and 3.

Initial choice of items to factor analyze

How did Radloff chose items?

. She collected a list of common depressive symptoms

. Is that a good approach?

How you might choose from among the 20 CES-D items

. Previous research findings

. Your own theory

. Needs of your particular research question

Respondent sampling and data collection (here, secondary data)

Compute matrix of inter-item correlations

Example: Step 4. specify number of factors

Options

. a priori choice: theory, prior empirical findings

. Eigenvalue > 1.0

. Scree plot

. Model fit tests, indices

Example: Step 4. Specify number of factors

a priori choice

. Many investigators, but not all, have reported 4 factors

Example: Step 4. Specify number of factors

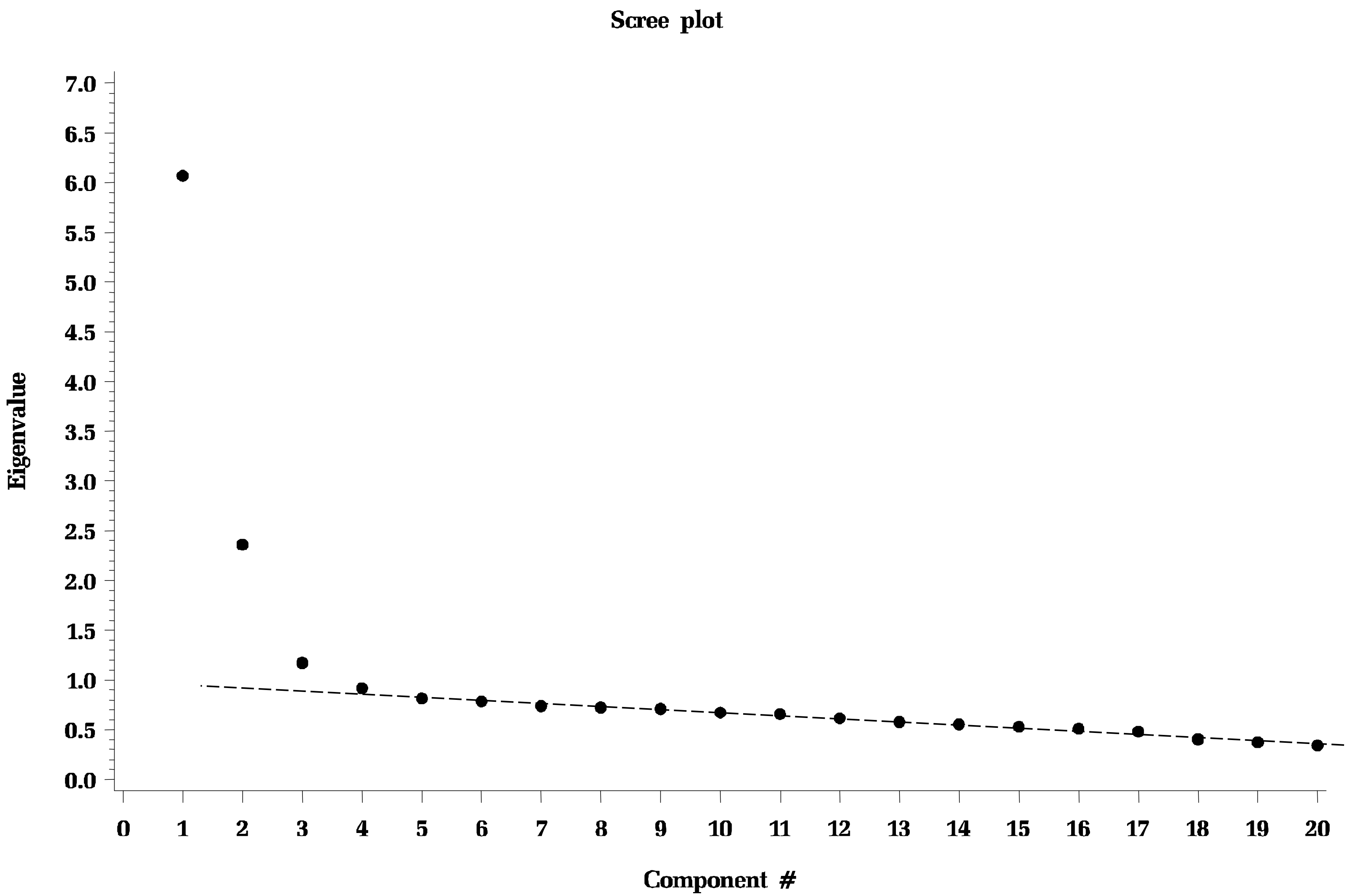
Choose number of factors to equal number of eigenvalues > 1.0

Eigenvalues of the full item correlation matrix

	Eigenvalue	Proportion	Cumulative
1	6.06974392	0.3035	0.3035
2	2.36134193	0.1181	0.4216
3	1.17053740	0.0585	0.4801
4	0.91717920	0.0459	0.5259
5	0.81328821	0.0407	0.5666
6	0.78490631	0.0392	0.6058
7	0.73721336	0.0369	0.6427
8	0.72236274	0.0361	0.6788
9	0.70879857	0.0354	0.7143
10	0.67176187	0.0336	0.7479
11	0.65731560	0.0329	0.7807
12	0.61666653	0.0308	0.8116
13	0.57798797	0.0289	0.8405
14	0.55424913	0.0277	0.8682
15	0.52913967	0.0265	0.8946
16	0.50941923	0.0255	0.9201
17	0.48174892	0.0241	0.9442
18	0.40293658	0.0201	0.9643
19	0.37224703	0.0186	0.9829
20	0.34115581	0.0171	1.0000

Example: Step 4. Specify number of factors

Scree plot



Example: Step 4. Specify number of factors

Goodness-of-fit chi-square and CFI

Only available with maximum likelihood (ML) EFA

There is a chi-square test that the number of factors is sufficient

. A significant test suggests more factors should be extracted

. Test is sensitive to departures from multivariate normality

There also are various indices of approximate fit, e.g., CFI

. CFI values range between 0 and 1

. CFI values > .95 are thought to suggest approximate fit

# of factors	χ^2	*df*	*p*	CFI
1	8226	170	<.0001	.77
2	3155	151	<.0001	.91
3	1941	133	<.0001	.95
4	1053	116	<.0001	.97
5	745	100	<.0001	.98

Example: Step 4. Specify number of factors

Summary

. Radloff and many others suggested 4 factors

. Eigenvalue > 1.0 suggested 3 factors

. Scree plot suggested 3 factors

. Model fit suggested at least 3 factors

We will consider models with 2 through 4 factors

Example: Steps 5 and 6

Specify method of factor extraction

I selected ML factor extraction

It allows for the tests/indices of model fit, described above

Specify method of factor rotation

I selected Harris-Kaiser (oblique) rotation

For comparison, I also present a VARIMAX (orthogonal) rotation

Example: Step 7: Assess the model

2 factors with Harris-Kaiser rotation

		(Negative) Factor1	(Pos Aff) Factor2
cesd06	depressed	77	.
cesd18	sad	72	.
cesd03	blues	70	.
cesd14	lonely	68	.
cesd07	effort	61	.
cesd10	fearful	60	.
cesd20	get going	59	.
cesd05	mind	59	.
cesd17	cry	56	.
cesd01	bothered	53	.
cesd09	failure	52	.
cesd11	restless	49	.
cesd19	dislike	48	.
cesd02	appetite	48	.
cesd13	talk	47	.
cesd15	unfriendly	42	.
cesd12	happy	.	76
cesd16	enjoy	.	76
cesd08	hopeful	.	60
cesd04	good	17	55

Example: Step 7: Assess the model

Inter-Factor Correlations

	Factor1	Factor2
Factor1	1.00	
Factor2	-0.30	1.00

Example: Step 7: Assess the model

3 factors

		(Dep+Som) Factor1	(InterPers) Factor2	(Pos Aff) Factor3
cesd06	depressed	82	.	.
cesd03	blues	75	.	.
cesd07	effort	70	-10	.
cesd01	bothered	66	-16	.
cesd20	get going	61	.	.
cesd18	sad	59	17	.
cesd05	mind	58	.	.
cesd02	appetite	57	-11	.
cesd14	lonely	54	18	.
cesd11	restless	52	.	.
cesd10	fearful	46	19	.
cesd17	cry	45	15	.
cesd13	talk	38	11	.
cesd19	dislike	.	74	.
cesd15	unfriendly	.	62	.
cesd09	failure	22	40	.
cesd12	happy	.	.	76
cesd16	enjoy	.	.	76
cesd08	hopeful	.	.	60
cesd04	good	24	.	56

Example: Step 7: Assess the model

Inter-Factor Correlations

	Factor1	Factor2	Factor3
Factor1	1.00		
Factor2	0.67	1.00	
Factor3	-0.30	-0.21	1.00

Example: Step 7: Assess the model

4 factors

		(Somatic) Factor1	(InterPers.) Factor2	(Pos Aff) Factor3	(Dep Aff) Factor4
cesd07	**effort**	**80**	.	.	**-11**
cesd20	**get going**	**74**	.	.	**-14**
cesd05	**mind**	**47**	.	.	**11**
cesd11	**sleep**	**44**	.	.	.
cesd02	**appetite**	**42**	.	.	**14**
cesd01	**bothered**	**35**	**-14**	.	**31**
cesd13	**talk**	**35**	**15**	.	.
cesd19	**dislike**	.	**69**	.	.
cesd15	**unfriendly**	.	**61**	.	.
cesd09	failure	16	39	.	10
cesd16	**enjoy**	.	.	**76**	.
cesd12	**happy**	.	.	**76**	.
cesd08	**hopeful**	.	.	**60**	.
cesd04	**good**	**11**	.	**55**	**13**
cesd18	**sad**	**-15**	.	.	**86**
cesd17	**cry**	**-27**	.	.	**84**
cesd06	**depressed**	**22**	.	.	**65**
cesd03	**blues**	**17**	.	.	**63**
cesd14	**lonely**	.	**11**	.	**60**
cesd10	**fearful**	**13**	**15**	.	**37**

Example: Step 7: Assess the model

Inter-Factor Correlations

	Factor1	Factor2	Factor3	Factor4
Factor1	1.00			
Factor2	0.54	1.00		
Factor3	-0.24	-0.18	1.00	
Factor4	0.82	0.63	-0.30	1.00

Example: Effects of other options

Unrotated factors: 4-factor model

		Factor1	Factor2	Factor3	Factor4
cesd06	depressed	77	.	-13	-11
cesd18	sad	73	.	.	-24
cesd03	blues	70	.	-11	-12
cesd14	lonely	67	.	.	-11
cesd07	effort	59	13	-23	26
cesd10	fearful	59	.	.	.
cesd20	get going	58	12	-15	27
cesd05	mind	56	12	.	13
cesd17	cry	56	.	.	-28
cesd01	bothered	53	.	-19	.
cesd09	failure	53	.	22	11
cesd19	dislike	49	.	47	13
cesd11	sleep	49	.	-11	12
cesd02	appetite	46	.	-16	.
cesd13	talk	44	10	.	13
cesd15	unfriendly	41	.	39	16
cesd16	enjoy	-32	69	.	.
cesd12	happy	-36	69	.	.
cesd08	hopeful	-19	55	.	.
cesd04	good	.	52	.	.

Example: Effects of other options: VARIMAX (orthogonal rotation)

		Factor1 (depr.+som.)	Factor2 (depr.)	Factor3 (pos. aff.)	Factor4 (int.pers)
cesd07	**effort**	**67**	**12**	.	**14**
cesd20	**get going**	**63**	**10**	.	**20**
cesd06	**depressed**	**57**	**52**	**-10**	**16**
cesd05	**mind**	**52**	**21**	.	**21**
cesd03	**blues**	**50**	**49**	.	**15**
cesd01	**bothered**	**48**	**29**	.	.
cesd11	**sleep**	**46**	**17**	.	**14**
cesd02	**appetite**	**45**	**19**	.	.
cesd10	**fearful**	**38**	**35**	.	**29**
cesd13	**talk**	**38**	**15**	.	**23**
cesd18	**sad**	**37**	**61**	.	**28**
cesd17	**cry**	**23**	**55**	.	**21**
cesd14	**lonely**	**40**	**48**	.	**29**
cesd12	**happy**	**-14**	**-13**	**75**	.
cesd16	**enjoy**	**-12**	.	**75**	.
cesd08	**hopeful**	.	.	**58**	.
cesd04	**good**	.	.	**52**	.
cesd19	**dislike**	**17**	**19**	.	**65**
cesd15	**unfriendly**	**18**	**12**	.	**56**
cesd09	**failure**	**31**	**22**	.	**43**

fuzzy separation of depressive affect and somatic symptom factors. 17 cross-loadings > .20

EFA Conclusions

Choice of number of factors should be based upon

Theoretical appeal, parsimony, clinical experience *as much as* empirical model fit

To explore the properties of multi-item measurement instruments…

. Use factor analysis, not principal components analysis

. Use oblique rotation

. Use ML factor analysis, which allows for consideration of CFI

. Be thoughtful about the measurement model

. Item creation and selection should be a deliberate process

EFA Conclusions

Creation/adaptation of a measurement instrument
with good psychometric properties
represents programmatic work—not project work.

Qualitative research methods in construct identification
and item development, e.g.,

. Focus groups and/or individual interviews

. Cognitive interviewing

Iterative refinement of instrument with new samples

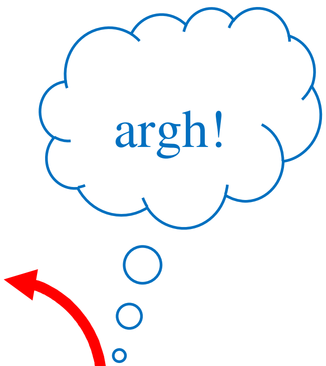
Replication and confirmation (confirmatory factor analysis: CFA)

Adapting and testing in new population groups

EFA frustrations

Exploratory factor analysis can be confusing/trying/squishy

Especially w/ large measurement models



Simultaneous challenge

. (i) determine which items to drop from consideration…
extraneous items can obfuscate factor structure

. (ii) if the number of extracted factors is incorrect, then
an important item can appear to be extraneous

. Many have sought a 'holy grail' rotation method—it doesn't exist

These challenges mostly are addressed by SAS PROC VARCLUS

(I will present on VARCLUS in second talk next month)

Thank you